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SLIDING MODE CONTROL OF MOTIONS
OF TOWED SHIPS

by

Arthur K. Samora

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Sliding Mode Control of Motions of Towed Ships

by

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ABSTRACT

A control system based on sliding mode control and the linear quadratic regulator is designed to stabilize the straight line motions of towed vessels. The control technique is through athwartship movement of the towline attachment point on the towed vessel. Control design is based on the linearized sway and yaw equations of motion. Numerical simulations for both the linearized and the nonlinear system are performed and demonstrate the added robustness of the control technique employed.

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I. INTRODUCTION

The horizontal plane stability of towed vessels is a very important field. If the motion of a vessel under tow is unstable, a very dangerous situation exists; the motion of a several thousand ton vessel could endanger the ship towing it [Refs. 1, 2, 3]. A method needs to be developed not only to ensure the stability of the towed vessel's motion, but to control it [Refs. 4, 5]. If the motion of the towed vessel can be controlled, the next step is to optimize its control, both for safety and fuel considerations. Usually the only means of control of a vessel under tow is through the towline. Under these conditions, stability of the vessel is marginal; in some cases, the vessel is actually unstable.

This thesis will develop the equations of motion for a vessel in tow. It will also state and describe the unique forces associated with a vessel being towed. Finally, it will describe how state-space theory is being applied to control the towed vessel's motion through the movement of the attachment point of the towline along the beam.

A control system based on sliding mode control and the linear quadratic regulator for the linearized equations of motion will be presented, with results in non-dimensional form. The same control system for the non-linearized equations of motion will also be presented, along with the results. Comparison of results for the linearized and non-linearized equations of motion will help determine the

robustness of the system. $Matrix_x$ will be used to calculate the required gains for the system, and the simulation will be done using a Fortran program.

II. EQUATIONS OF MOTION

The two equations of motion will be developed using a body-fixed frame of reference. The origin of this reference frame is at the center of gravity of the towed vessel. In general, the vessel has six degrees of freedom:

1. surge along the x_g-axis
2. roll about the x_g-axis
3. sway along the y_g-axis
4. pitching about the y_g-axis
5. heave along the z_g-axis
6. yaw about the z_g-axis

This thesis will only deal with motion in the horizontal plane. Also, a constant surge velocity along the x_g-axis will be assumed. Other assumptions include:

1. no towed vessel/towing vessel interaction
2. massless, inextensible towline
3. no wind, wave, or current disturbances
4. the body is symmetric

Since motion is confined to the horizontal plane, the applicable equations are the sway and yaw equations. These equations are standard Principles of Naval Architecture, or PNA, equations. They are as follows:

Sway:

$$m\,[\,\dot{v} + ur + wp + x_g(pq + \dot{r}) - y_g(p^2 + r^2) + z_g(qr - \dot{p})\,]$$
$$= (W - B)\cos\theta\cos\phi + Y_f \qquad \textbf{(1a)}$$

Yaw:

$$I_z\dot{r} + (I_y - I_x)_{pq} - I_{xy}(p^2 - q^2) - I_{yz}(pr + \dot{q}) +$$
$$m\,[\,x_g(\dot{v} + ur + w) - y_g(\dot{u} - vr + wq)\,] \qquad \textbf{(1b)}$$
$$= (x_g w - x_\beta B)\cos\theta\cos\phi + (y_g W - y_\beta B)\sin\theta + N_f$$

If we make the above assumptions in terms of the variables in the equations:

1. No vertical motion; w=0
2. W=B
3. Small angle motion (linearized simulation only)
4. No pitch rate; also negligible roll; $\theta=0$, $\dot{\theta}=0$, $\phi=0$
5. Forward speed equals nominal speed; u=U
6. Center of Buoyancy=Center of gravity; x_g, y_g=x_β, y_β

Then the sway and yaw equations become:

$$m\,(\dot{v} + ur + x_g\dot{r}) = Y_f \qquad \textbf{(2a)}$$

$$I_z\dot{r} + mx_g\,(\dot{v} + ur) = N_f \qquad \textbf{(2b)}$$

The hydrostatic forces on the right hand side of the equations are then expanded in a standard Taylor series:

$$Y_F = Y_v v + Y_r r + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{towline} \qquad \textbf{(3a)}$$

$$N_F = N_v v + N_r r + N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{towline} \qquad \textbf{(3b)}$$

where $\quad Y_v = \frac{\partial Y}{\partial v}\Big|_{v=0}$, etc.

Note that the rudder terms, N_δ and Y_δ have been omitted, as the towed vessel's rudder is amidships; there is no applied force due to the rudder. The only applied force is due to the towline. Substituting (3a) into (2a) and (3b) into (2b) and rearranging the terms yields:

Sway:

$$(m - Y_{\dot{v}})\,\dot{v} + (-Y_{\dot{r}})\,\dot{r} = Y_v v + (Y_r - mu)r + Y_{towline} \qquad \textbf{(4a)}$$

Yaw:

$$(I_z - N_{\dot{r}})\dot{r} - N_{\dot{v}}\dot{v} = N_v v + N_r r + N_{towline} \qquad \textbf{(4b)}$$

which can then be rearranged into working form as

$$Y_v v + (Y_{\dot v} - m)\dot v + (Y_{\psi} - mu)\dot\psi + Y_{\ddot\psi}\ddot\psi = Y_{towline} \tag{5a}$$

$$N_v v + N_{\dot v}\dot v + N_{\psi}\dot\psi + (N_{\ddot\psi} - I_z)\ddot\psi = N_{towline} \tag{5b}$$

where $r=\dot\psi$, $\dot r=\ddot\psi$. All that remains is to describe the towline forces in greater detail.

A. TOWLINE FORCE

In Figure 1, P is the attachment point of the tow. x_p and y_p denote the distance from the towed vessel's center of gravity, again using the standard PNA body-fixed coordinate system. From the diagram, we can see that the horizontal force associated with the towline in the sway direction is:

$$T\sin(\gamma + \psi)$$

The moment causing the vessel to yaw is:

$$-Tx_p(\sin(\gamma + \psi)) - Ty_p\cos(\gamma + \psi)$$

Assuming γ and ψ are small angles:

$$\sin(\gamma + \psi) = \gamma + \psi = \frac{y + x_p\psi + y_p}{L} + \psi$$

$$= \psi\left(1 + \frac{x_p}{L}\right) + \frac{y}{L} + \frac{y_p}{L}$$

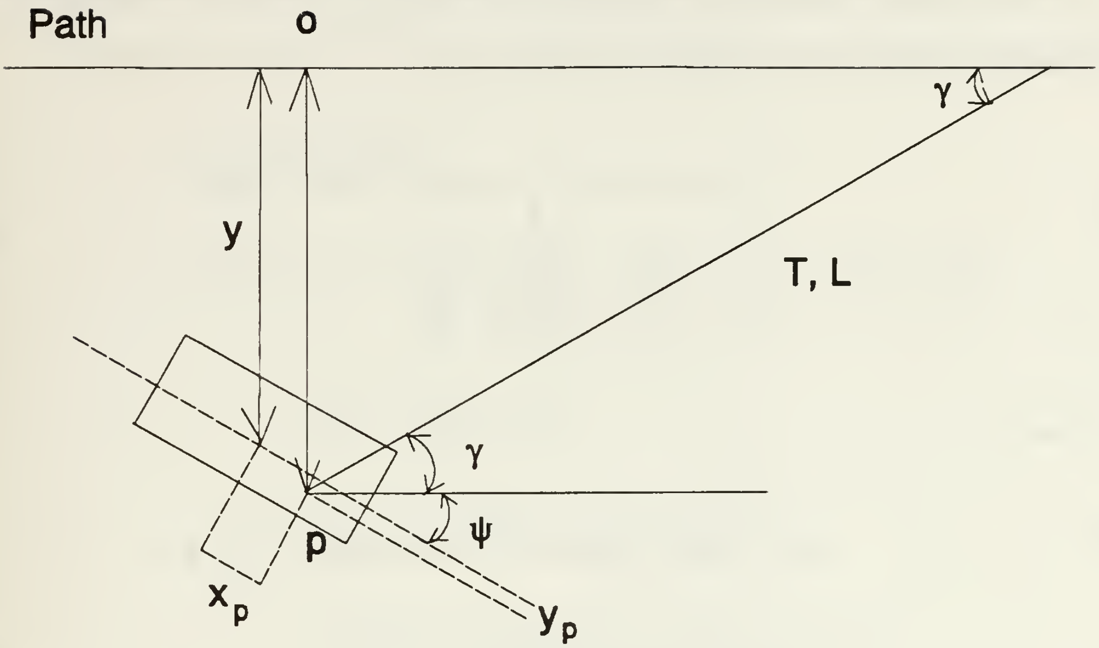

Figure 1. Towed Vessel Coordinate Description.

Also,

$$\dot{y} = v + u\sin\psi = v + u\psi$$

$$v = \dot{y} - u\psi\,;\ \dot{v} = \ddot{y} - u\ddot{\psi}$$

Finally, substituting for v, in equations (5a) and (5b) and rearranging terms:

Sway:

$$
\begin{aligned}
&(Y_{\dot v} - m)\ddot y + Y_{v}\dot y - \frac{T}{L} y + Y_{\dot\psi}\ddot\psi - (Y_{\dot v} u - Y_{\psi})\dot\psi \\
&- [Y_{v} u + T \left(1 + \frac{x_p}{L} \right) \psi = \frac{T y_p}{L}
\end{aligned}
\tag{6a}
$$

Yaw:

$$
\begin{aligned}
&N_{\dot v}\ddot y + N_{v}\dot y - \frac{T}{L} x_p y + (N_{\psi} - I_z)\ddot\psi - (N_{\dot v} u - N_{\psi})\dot\psi \\
&- [N_{v} u + T x_p \left(1 + \frac{x_p}{L} \right)] \psi \\
&= \frac{T}{L} x_p y_p + T y_p
\end{aligned}
\tag{6b}
$$

These two equations are motion describe the towed vessel's motion. Note that the coordinate y describes the lateral distance from the towing path; y_p describes the distance of the towline attachment point from the centerline of the towing vessel. It is for y_p that the control law is being developed.

B. ARRANGEMENT INTO STATE-SPACE

Let $x_1=y$, $x_2=\psi$, $x_3=\dot{y}$, $x_4=\dot{\psi}$, then, substituting into equations (6a) and 6b)

$$(Y_{\dot{v}} - m)\dot{x}_3 + Y_{\dot{\psi}} x_4$$

$$= \frac{T}{L}x_1 + [Y_v u + T\left(1 + \frac{x_p}{L}\right)]x_2 - Y_v x_3 + (Y_{\dot{v}} u - Y_\psi)x_4 + \frac{T}{L}y_p \tag{7a}$$

$$N_v x_3 + (N_{\ddot{\psi}} - I_z)\dot{x}_4$$

$$= \frac{T}{L}x_p x_1 + [N_v u + Tx_p\left(1 + \frac{x_p}{L}\right)] \tag{7b}$$

$$x_2 - N_v x_3 + (N_{\dot{v}} u - N_\psi)x_4 + \frac{T}{L}x_p y_p + Ty_p$$

From the definition of the state variables, it can be seen that

$$\dot{x}_1 = x_3 \tag{8a}$$

and

$$\dot{x}_2 = x_4 \tag{8b}$$

These comprise the first two of four state equations. The second two state equations must come from equations (7a) and (7b) above. Note that these equations are coupled in x_3 and x_4. Algebraic elimination yields the final two equations of motion. Multiplying equation (7a) by $(N_\psi - I_z)$ and equation (7b) by $(-Y_\psi)$ yields an equation for $\dot{x}_3$.

$$
\begin{aligned}
&[\,(N_{\dot\psi} - I_z)(Y_{\dot v} - m) - N_{\dot v}\, y_{\dot\psi}\,]\,\dot{x}_3 \\
&= \frac{T}{L}[\,(N_{\dot\psi} - I_z) - x_p Y_{\dot\psi}\,]\,x_1 \\
&+ \left\{ \left[Y_v u + T\left(1 + \frac{x_p}{L}\right)\right](N_{\dot\psi} - I_z) \right. \\
&\left. - Y_{\dot\psi}\left[N_v u + T x_p\left(1 + \frac{x_p}{L}\right)\right] \right\} x_2 \\
&+ [\,N_v Y_{\dot\psi} - Y_v(N_{\dot\psi} - I_z)\,]\,x_3 \\
&+ [\,(Y_{\dot v} u - Y_\psi(N_{\dot\psi} - I_z) - Y_{\dot\psi}(N_{\dot v} u - N_\psi\,]\,x_4 \\
&+ T\left[\frac{(N_{\dot\psi} - I_z)}{L} - Y_{\dot\psi}\left(1 + \frac{x_p}{L}\right)\right] y_p
\end{aligned}
\tag{8c}
$$

Similarly, multiplying equation (7a) by $(-N_{\dot v})$ and equation (7b) by $(Y_{\dot v} - m)$ and adding yields an equation for $\dot{x}_4$.

$$
\begin{aligned}
&[\,(N_{\ddot{\psi}} - I_z)(Y_{\dot{v}} - m) - N_{\dot{v}}\, y_{\ddot{\psi}}\,]\,\dot{x}_4 \\
&= \frac{T}{L}\,[\,x_p (Y_{\dot{v}} - m) - N_{\dot{v}}\,]\,x_1 \\
&+ \{ (Y_{\dot{v}} - m)\left[N_v u + T x_p \left(1 + \frac{x_p}{L} \right) \right] \\
&- N_{\dot{v}} \left[Y_v u + T \left(1 + \frac{x_p}{L} \right) \right] \} \, x_2 \\
&+ [\, N_{\dot{v}} Y_v - N_v (Y_{\dot{v}} - m)\,]\, x_3 \\
&+ [\, -N_{\dot{v}}\, m u + N_{\psi} m - N_{\psi} Y_{\dot{v}} + N_{\dot{v}}\, y_{\psi}\,)\, x_4 \\
&+ T \left[(Y_{\dot{v}} - m) \left(1 + \frac{x_p}{L} \right) - \frac{N_{\dot{v}}}{L} \right] y_p
\end{aligned}
\tag{8d}
$$

Equations (8a), (8b), (8c) and (8d) can also be expressed in state-space matrix form

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}
=
\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ b_3 \\ b_4 \end{bmatrix} y_p
\tag{9}
$$

Where,

$$D = (N_{\psi} - I_z)\,(Y_v - m) - N_v Y_{\psi}$$

$$a_{31} = D^{-1} \frac{T}{L}\,[\,(N_{\psi} - I_z) - x_p Y_{\psi}\,]$$

$$A_{32} = D^{-1} \left\{ \left[Y_v u + T \left(1 + \frac{x_p}{L} \right) \right] (N_{\psi} - I_z) - Y_{\psi} \left[N_v u + T x_p \left(1 + \frac{x_p}{L} \right) \right] \right\}$$

$$a_{33} = D^{-1} \left[N_v Y_\psi - Y_v \left(N_\psi - I_z \right) \right]$$

$$a_{34} = D^{-1} \left[\left(Y_v u - Y_\psi \right) \left(N_\psi - I_z \right) - Y_\psi \left(N_v u - N_\psi \right) \right]$$

$$a_{41} = D^{-1} \frac{T}{L} \left[x_p \left(Y_v - m \right) - N_v \right]$$

$$a_{42} = D^{-1} \left\{ \left(Y_v - m \right) \left[N_v u + T x_p \left(1 + \frac{x_p}{L} \right) \right] - N_v \left[Y_v u + T \left(1 + \frac{x_p}{L} \right) \right] \right\}$$

$$a_{43} = D^{-1} \left[N_v Y_v - N_v \left(Y_v - m \right) \right]$$

$$a_{44} = D^{-1} \left[N_v Y_\psi - N_\psi Y_v + \left(N_\psi - N_v u \right) m \right]$$

$$b_3 = D^{-1} T \left[\frac{\left(N_\psi - I_z \right)}{L} - Y_\psi \left(1 + \frac{x_p}{L} \right) \right]$$

$$b_4 = D^{-1} T \left[\left(Y_v - m \right) \left(1 + \frac{x_p}{L} \right) - \frac{N_v}{L} \right]$$

III. CONTROL SYSTEM DESIGN

A. SYSTEM AUGMENTATION

Equation (9) defines the state-space form of the equations of motion. One more state must be defined and added to this equation; that of the ordered, or commanded, y_p. This variable shall be called y_{pc}. The usual response of y_p with respect to time should look like a first order system, as in Figure 2:

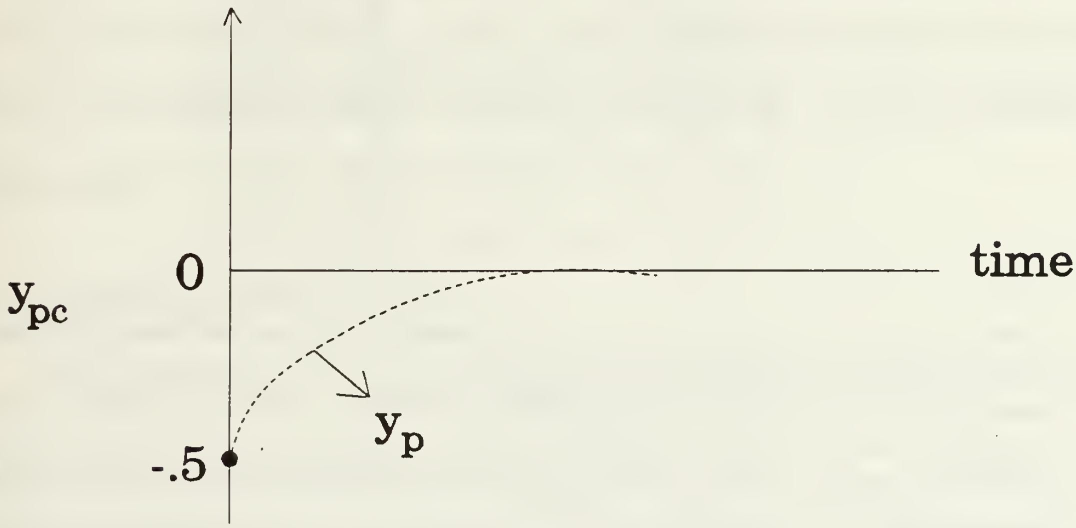

Figure 2. y_p Response with Respect to Time.

The equation governing this response is as follows:

$$T_p \dot{Y}_p + y_p = Y_{pc}$$

$$\text{or} \quad \dot{y}_p = \frac{-1}{T_p} y_p + \frac{1}{T_p} y_{pc}$$

where T_p is the non-dimensional time constant for the towline attachment point motion control system. If y_{pc} is now considered the input to the system, and y_p is considered a state variable, the matrix equation becomes:

$$\begin{bmatrix} \dot{y}_p \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{-1}{T_p} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ b_3 & a_{31} & a_{32} & a_{33} & a_{34} \\ b_4 & a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} y_p \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \frac{1}{T_p} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} y_{pc} \tag{10}$$

where a_{31}, a_{32}, a_{33}, a_{34}, a_{41}, a_{42}, a_{43}, a_4, b_3 and b_4 are as previously defined. In a sense, the actual distance of the towline attachment point y_p is treated as an extra state to the system. The control system will be designed based on this augmented system of equations. The control system will be designed based on two principles of state-space theory; Linear Quadratic Regulator, and Sliding Mode Control.

B. LINEAR QUADRATIC REGULATOR

The linear quadratic regulator function arose from efforts to find an optimum means of control [Ref. 6]. Normally, a gain matrix for a particular control system is calculated to achieve specific closed-loop pole locations. With

the linear quadratic regulator, or LQR, a specific performance criterion J (or "cost function") is defined, with the only stipulations on the poles being that they be negative, or stable. This criterion J is defined as:

$$J = \int [x'(t)Q(t)x(t) + u'(t)R(t)u(t)]\,dt$$

The matrices Q and R are weighting matrices. The matrix Q is a state weighting matrix, and R is a control weighting matrix. The gains for a control system can be calculated based on the defined matrices Q and R. For instance, if the elements of the matrix Q are small relative to R, the system will tolerate large errors in the final state with very little control effort. Conversely, if Q is large relative to R, very small errors in the state X will result, but with considerable control effort.

C. SLIDING MODE CONTROL

The second aspect of state-space theory to be used in controlling the towed vessel is that of sliding mode control. Since the control law is based on a linearized set of equations of motion, a lot of uncertainty in the response exists. Also, some of the parameters may vary. A good example for the towed vessel system is the towline tension T, which will certainly vary with time. A control law needs to be developed that will take into account both the uncertainties in the parameters, and any dynamics that have either not been modeled, or that have been linearized. A sliding mode control law can ensure both stability and robustness of the system, with the emphasis on robustness. The LQR gains will

ensure stability of the system. Sliding mode control is ideally suited to a system where the response oscillates between set values, such as the motion of a towed vessel; it uses a high speed "toggling" control law to drive the system onto a desired "sliding plane."

Sliding mode control takes the standard state-space system

$$\dot{x} = [A]x + [b]u \tag{12}$$

and defines a sliding plane

$$\theta(x) = s_1 x_1 + s_2 x_2 + s_3 x_3 + s_4 x_4 \tag{13}$$

and the coefficient s_1 is arbitrary. Equation (13) can be written as

$$s^T x = 0$$

where

$$s^T = [s_1, s_2, s_3, s_4]$$

Next, define the Lyapunov function

$$V(x) = \frac{1}{2}[\theta(x)]^2$$

stability is guaranteed, provided $\dot{V}(x)$ is a negative definite function. Another way to express this is

$$\dot{V}(x) = \theta(x)\dot{\theta}(x) = -\eta^2 \theta(x) \tag{14}$$

therefore,

$$\dot{\theta} = -\eta^2 sign\theta \tag{15}$$

Since

$$\theta(x) = s^T(x), \dot{\theta}(x) = s^T(\dot{x}) = s^T(Ax + bu)$$

so, substituting for θ in Equation (15):

$$s^T(Ax + bu) = -\eta^2 sign(\theta)$$

and solving for u:

$$u = -(s^T b)^{-1} s^T Ax - \eta^2 (s^T b)^{-1} sign(\theta) \tag{16}$$

Equation (16) is a sliding mode control law for a generic system.

D. THE CONTROL LAW

All that remains is to define a control law for the augmented equations of motion for the towed vessel. The control law is defined using sliding mode control and linear quadratic regulator principles. Take the augmented system:

$$\begin{array}{c} \qquad\qquad A_{11} \qquad\quad A_{12} \\ \begin{bmatrix} y_p \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{T_p} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ b_3 & a_{31} & a_{32} & a_{33} & a_{34} \\ b_4 & a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} y_p \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \frac{1}{T_p} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} y_{pc} \\ \qquad\qquad A_{21} \qquad\quad A_{22} \end{array} \quad \mathbf{(17)}$$

and split the augmented matrix A into A_{11}, A_{12}, A_{21}, and A_{22} as shown. Also split the state vector into

$$y_1 = y_p \, ; \; y_2^T = [\, x_1, x_2, x_3, x_4 \,]$$

Rewrite equation (17) as

$$\dot{y}_1 = A_{11} y_1 + A_{12} y_2 + b_1 u$$

$$\dot{y}_2 = A_{21} y_1 + A_{22} y_2 + b_2 u$$

The original augmented state vector x now equals (y_1, y_2).

Next, define the sliding plane for the towed vessel as

$$\theta(x) = \theta(y_1, y_2) = y_1 + s^T y_2 = 0 \quad \mathbf{(18)}$$

and

$$\dot{\theta} = -\eta^2 sign(\theta) = \dot{y}_1 + s^T \dot{y}_2 \quad \mathbf{(19)}$$

but

$$\dot{y}_1 = A_{11}y_1 + b_1 u \quad \text{and} \quad \dot{y}_2 = A_{21}y_1 + A_{22}y_2$$

so, substituting for $\dot{y}_1$ and $\dot{y}_2$ is equation (19):

$$A_{11}y_1 + b_1 u + s^T(A_{21}y_1 + A_{22}y_2) = -\eta^2 sign(\theta)$$

solving for u:

$$u = \frac{-1}{b_1}[(A_{11} + s^T A_{21})y_1 + s^T A_{22}y_2] - \frac{1}{b_1}\eta^2 sign(\theta) \qquad \textbf{(20)}$$

This is the system control law for the towed vessel.

The gains [s] for the sliding mode control law from minimization of the linear quadratic regulator cost function J. Recall that

$$J = \int \{[x]'[Q][x] + [u]'[R][u]\} dt \qquad \textbf{(21)}$$

in Equation (21) the state weighting matrix Q is as follows:

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & q_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where q_{22} represents an error in radians from the path of the towing vessel. We are choosing to weigh the state variable $x_2 = \psi$. The control weighting matrix R will be defined by

$$R = (\alpha)^{-2}$$

where α, the maximum non-dimensional distance y_p will be weighted. With Q and R defined, the cost matrix function becomes:

$$J = \min \int (q_{22} x_2^2 + R u^2) dt \tag{22}$$

or

$$J = \min \int (q_{22} \psi^2 + R y_p^2) \, dt \tag{23}$$

The smaller the value for R is, the smaller the control effort required, but a larger state error in q_{22} will be have to be accepted as a trade-off. Conversely, the larger the value for R is the more control required, with very little state error in q_{22}.

The resulting gains are then placed into the control law for the augmented system of equations, Equation (20), and the response of the system is obtained.

IV. RESULTS

A. MATRIXX AND FORTRAN IMPLEMENTATION

MatrixX is an outstanding tool for understanding the response of a control system. A Fortran program was written to use with MatrixX. Since the coefficients of the [A] and [B] matrices of the state-space equation are constant with time, they can be put directly into the MatrixX software, and the resulting Linear Quadratic Regulator gains are easily calculated. Without the use of MatrixX, solving the linear quadratic integral is a very difficult problem in numerical analysis. The actual simulation of the system response was accomplished with a Fortran program on the VAX computer. The LQR gains obtained from MatrixX are put into a data file, along with the non-dimensional parameters of T_p, LL, X_p, T, and the initial conditions of Y, ψ, and the maximum distance Y_p can travel athwartships. The simulation then reads this data file, computes the sliding mode control and plots the response of the parameters Y, Y_p, ψ, and Y_{pc} with respect to time.

B. INITIAL CONDITIONS AND CONSTANT PARAMETERS

The results are presented graphically, and are in non-dimensional form, with the exception of the yaw angle ψ, which is in degrees. The non-dimensionalization is standard Principles of Naval Architecture: the relationships describing the

nondimensional parameters are included in the Appendix. Two vessels were studied:

1. 528 foot mariner
2. 1066.3 foot tanker

A summary of the hull particulars and hydrodynamic derivatives for both these vessels in also included in the Appendix. Four knots was used as the nominal forward velocity. The mariner is stable at four knots; the tanker is unstable. The towline tension for both vessels is taken from resistance curves at four knots, and then non-dimensionalized; the value used is 0.001 X_p, the longitudinal distance from the towed vessel center of gravity to the towline attachment point, is assumed to be constant. A value of 0.5 is used. The time required for Y_p to "match" Y_{pc} as in Figure 2, T_p is set as 0.5. The maximum non-dimensional distance Y_p can travel from port to starboard extremes is taken to be 0.1; therefore the maximum distance from an extreme to centerline is 0.05. The initial conditions for each data run are $Y=0.5$, $\psi=0$, and $Y_p=0.05$ as in Figure 3. Also, the weighting matrices were set at $q_{22}=5°$ (or .087 radians) for Q, the state error weighting matrix, and $\alpha=0.015$ for R, the control effect weighting for Y_{pc}. Four plots were generated for each simulation. The y-distance from the path (Y), the towline attachment point lateral offset (Y_p), the towline attachment point offset command (Y_{pc}) and the yaw angle (ψ) are all plotted vs nondimensional time. Variance of parameters of interest are shown on subsequent runs.

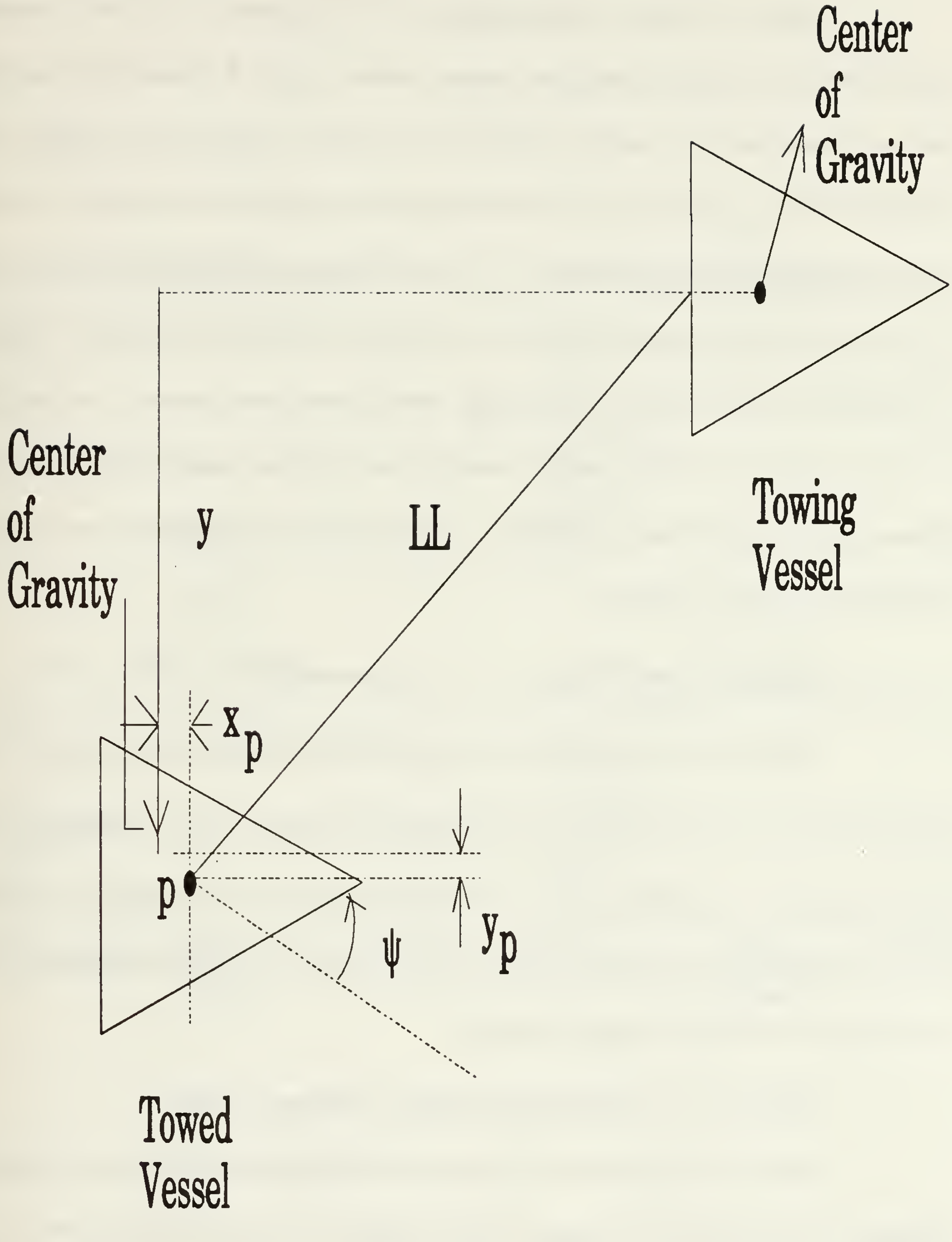

Figure 3. Initial Conditions.

1. Figures 4 and 5: Standard Mariner and Tanker

Figure 4 shows the response of the mariner with the "standard" parameters and initial conditions stated above; Figure 5 shows the "standard" tanker. Overshoot in Y and ψ exists for both cases, although it is higher in the mariner. The Y_{pc} graph shows that significantly more effort is required to control the tanker, and the oscillation in Y_p is much greater than for the mariner. This makes sense, as the tanker is inherently more unstable. Both vessels converge to a straightline path in approximately 60 non-dimensional seconds. As the tanker is more unstable, it will be used to demonstrate all other variations in parameters. Henceforth all references to "normal" or "standard" tanker response will be to the response shown in Figure 5.

2. Figure 6: Tanker with Y, $\dot{Y}$ Not Observable

Figure 6 shows the vessel response if the state variables Y and $\dot{Y}$ are not observable and are not used in the control law. Both Y and $\dot{Y}$ converge but the time required is significant; over 100 seconds. In this case there is no overshoot in Y, but considerable overshoot in ψ. ψ, Y_{pc}, and Y_p all oscillate considerably; but in the end, the system does converge.

3. Figures 7, 8: Observable Tanker, Variance of $\theta(x)$

Recall from Equation (13) that the sliding plane $\theta(x)$ is in part arbitrarily defined, as the gain η^2 is chosen by the designer. For the standard runs, η^2 was set to be 1.0. Figure 7 shows the tanker response if η^2 in $\theta(x)$ is set to 0.5, with all

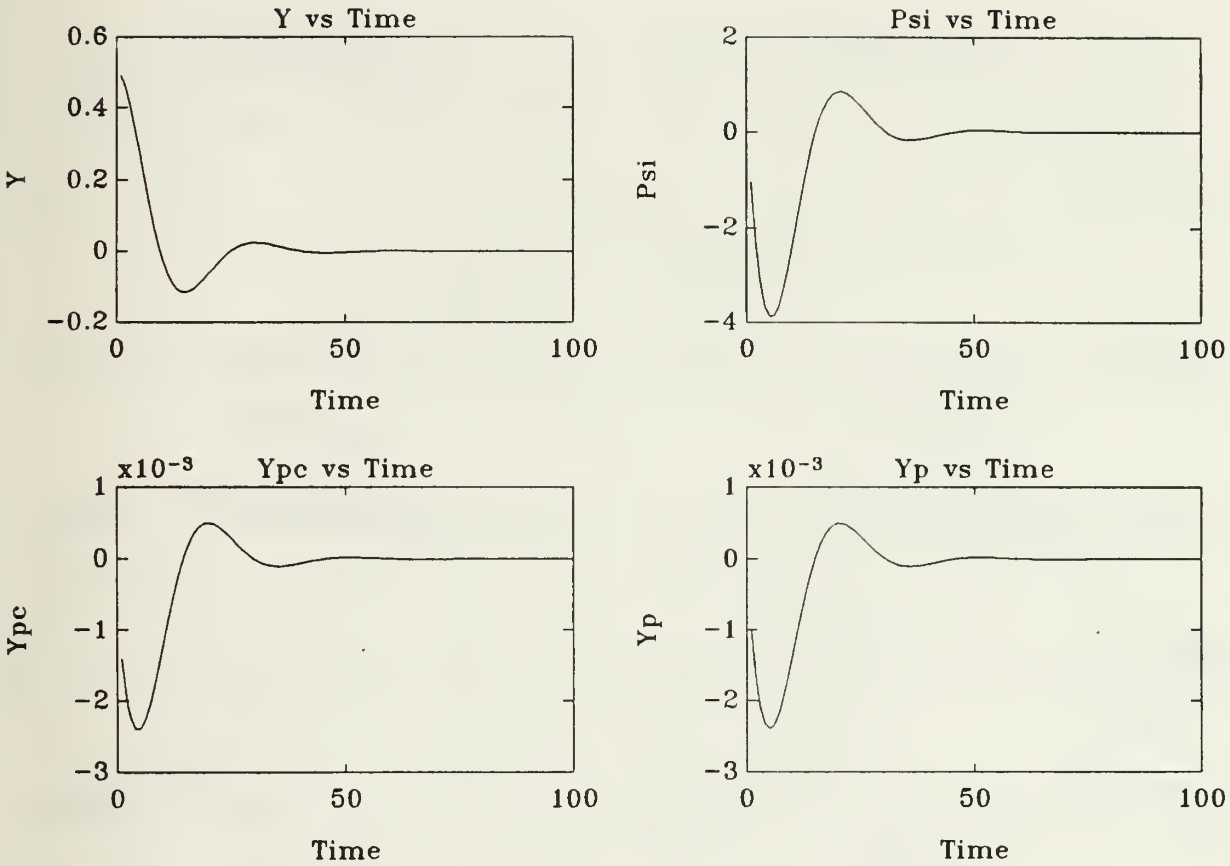

Figure 4. Mariner Response with Standard Parameters.

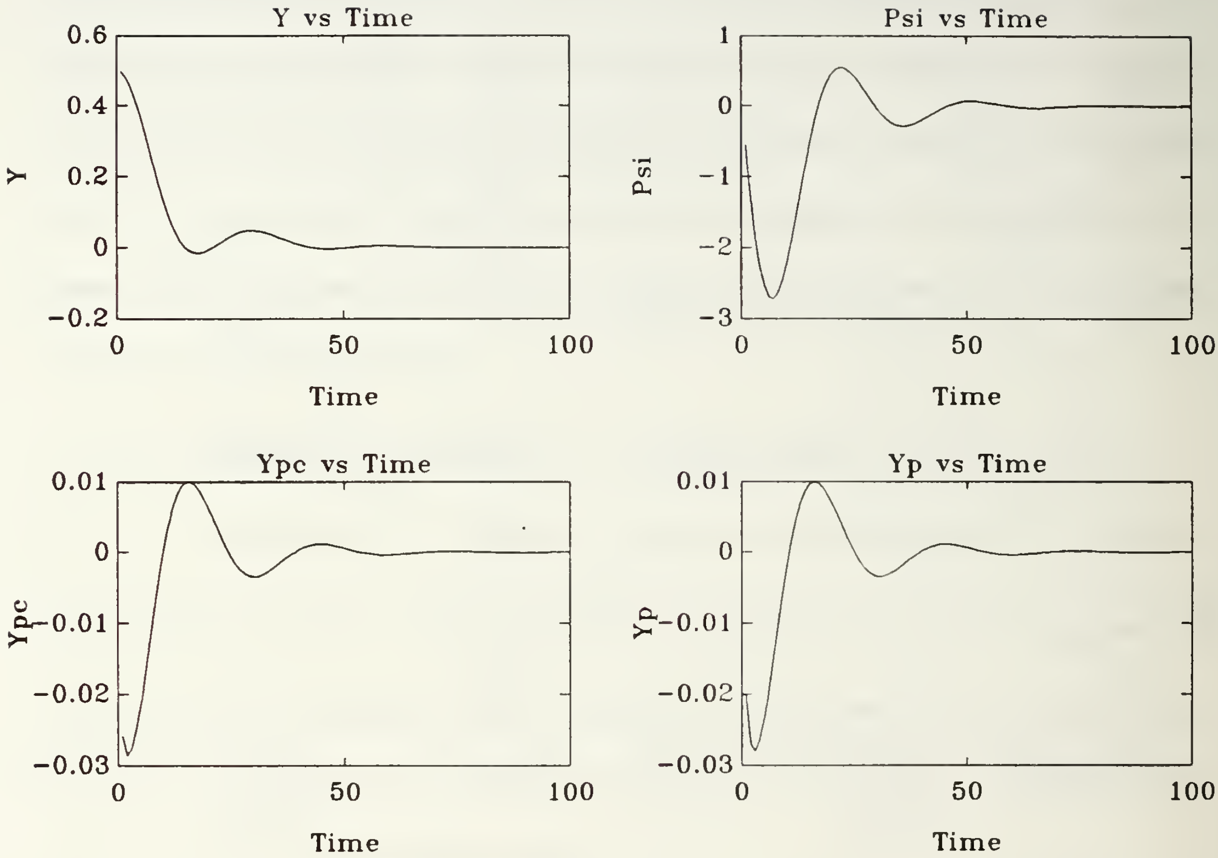

Figure 5. Tanker Response with Standard Parameters.

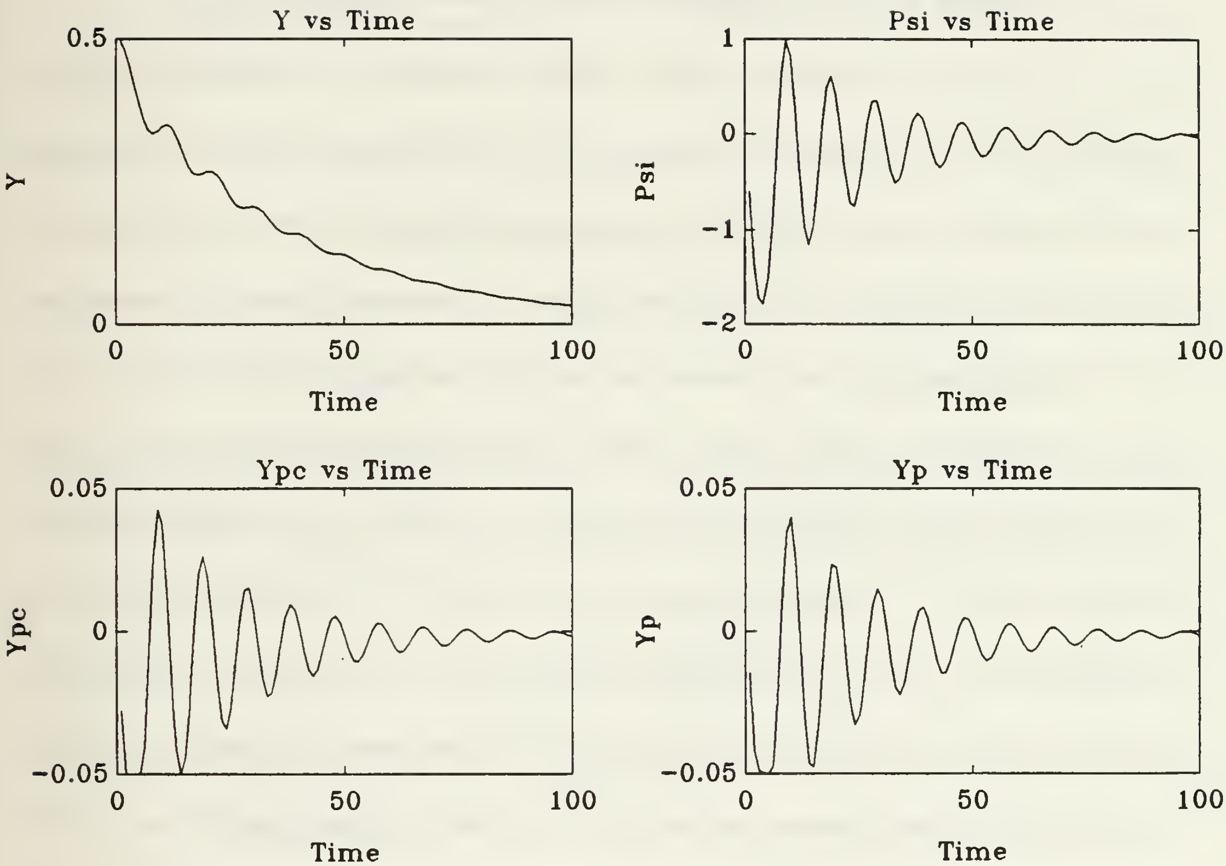

Figure 6. Tanker with Y, $\dot{Y}$ Not Observable.

state variables observable. The system still converges in adequate time; the parameters Y, Y_{pc}, and Y_p behave as in the standard tanker. There is approximately twice as much overshoot in ψ, however. In Figure 8, η^2 is set to 2.0; this brought the overshoot in ψ down to the level of the standard observable tanker, without altering the favorable response of the other three parameters.

4. Figures 9, 10, 11: Non-Observable Tanker, Variance of $\theta(x)$

The state variables Y and $\dot{Y}$ were assumed to be not observable in Figures 9 and 10. In Figure 9, η^2 was set to 0.5, with disastrous results. The towed vessel becomes unstable. All four parameters diverge with time. In Figure 10, however, η^2 in $\theta(x)$ is set to 2.0, and the system response stabilizes. The overshoot in ψ is approximately as the same as the standard tanker, but the overshoot in Y_p and Y_{pc} is twice that of the normal tanker. The settling time in Y is double that of the normal tanker, but the settling time in, Y_{pc} and Y_p is cut to forty seconds. Obviously there is a value between η^2 = 0.5 and η^2 = 1.0 in $\theta(x)$ where the response turns unstable for the non-observable case. In subsequent simulations, η^2 is set to be 2.0. Figure 11 summarizes the response of Y and Y_p for the observable and non-observable, with η^2 = 2. The top two graphs are for the observable case, and the bottom two describe the non-observable case.

5. Figure 13: Variance of the Sliding Plane Switching

Since sliding mode is a high-speed switching, or toggling method or control, the speed at which the switching is to be accomplished must be defined.

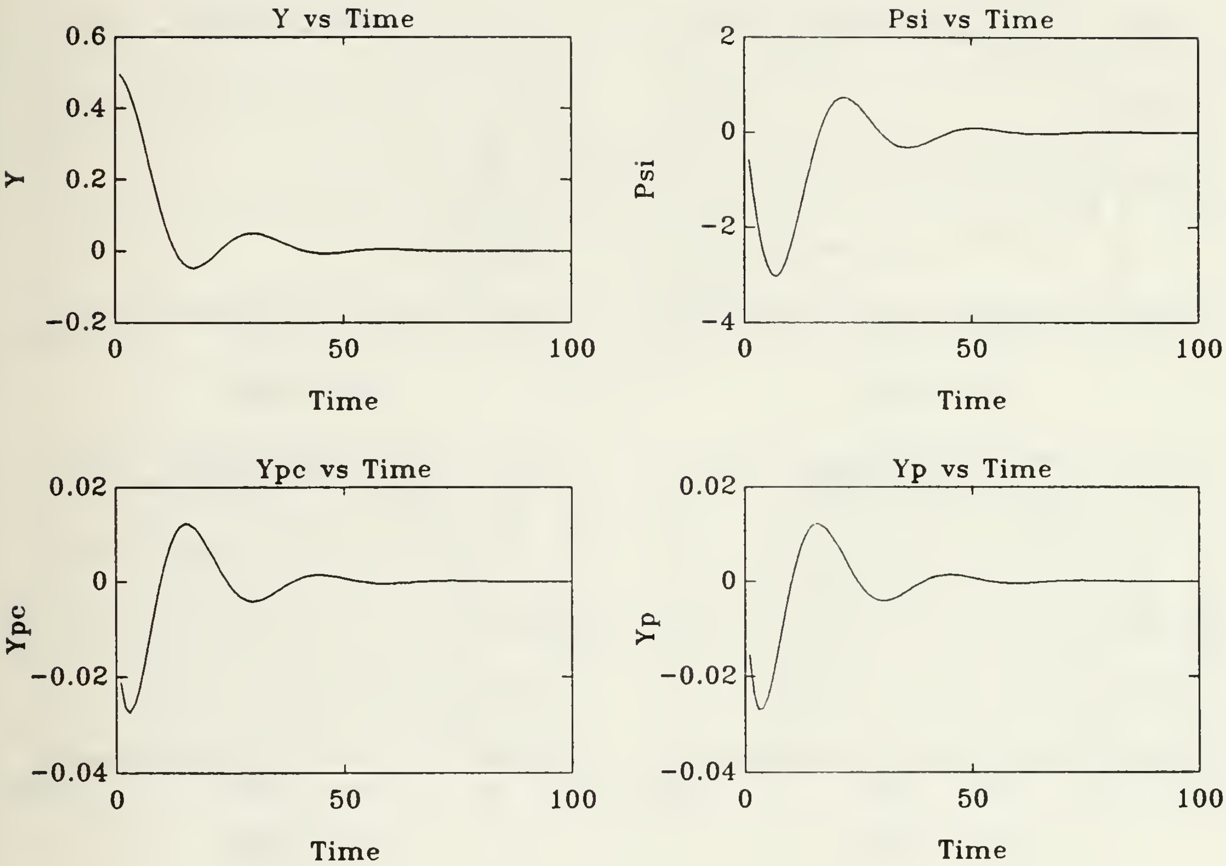

Figure 7. Observable Tanker, η^2=0.5.

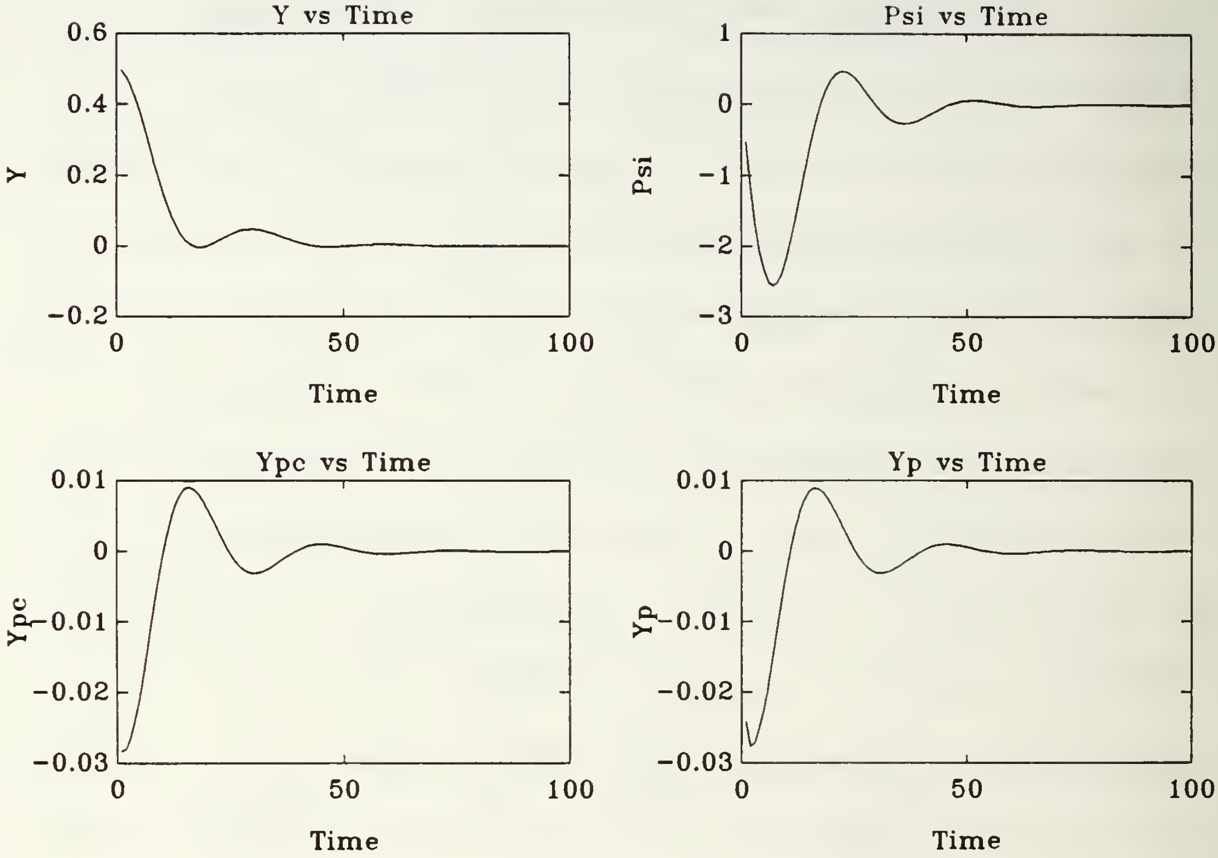

Figure 8. Observable Tankers, η^2=2.0.

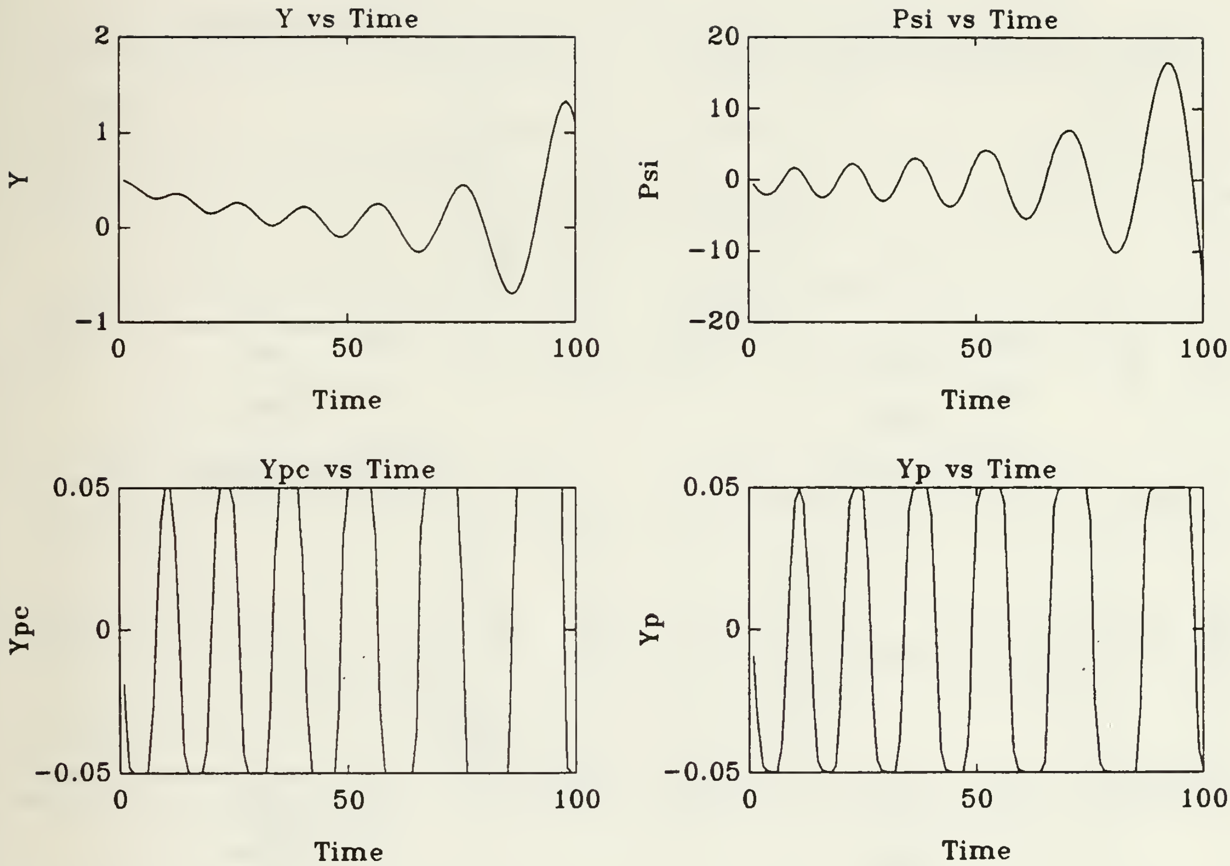

Figure 9. Non-Observable Tanker, η^2=0.5.

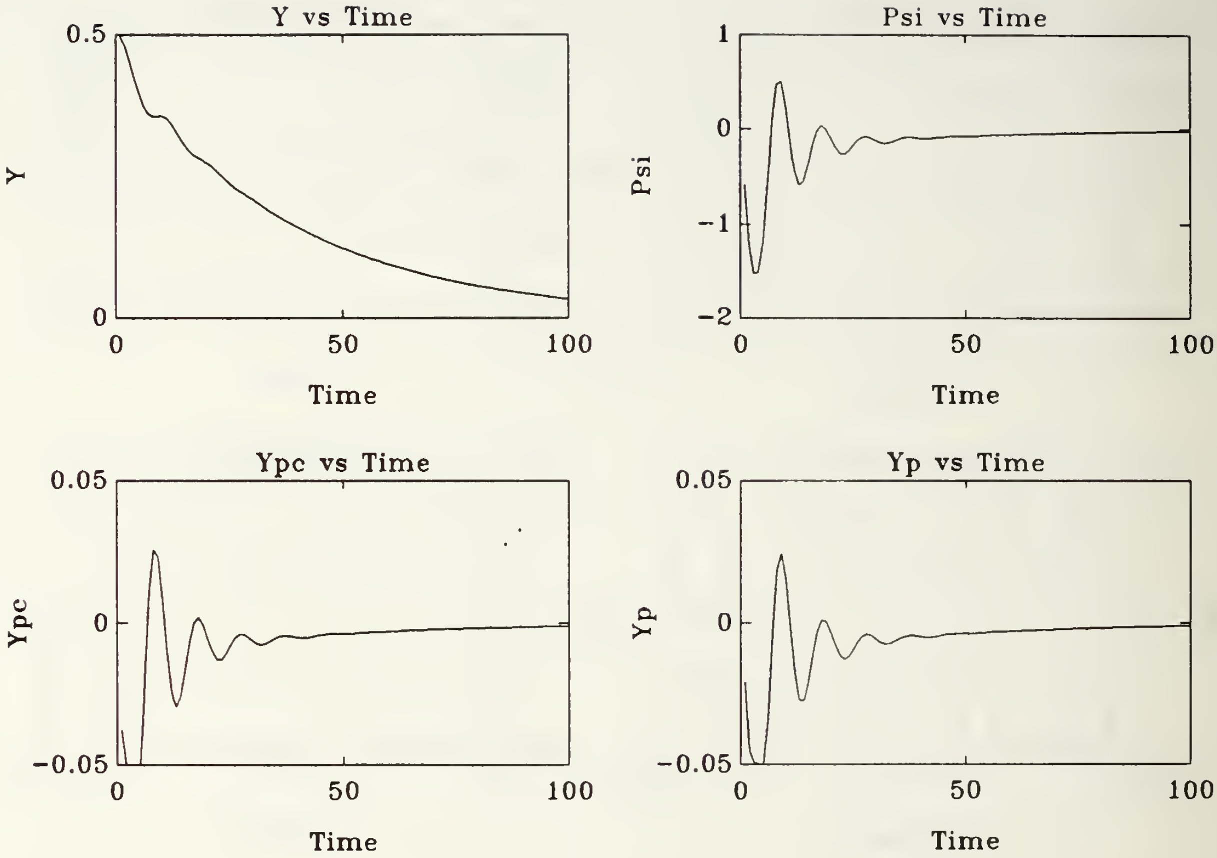

Figure 10. Non-Observable Tanker, η^2=2.0.

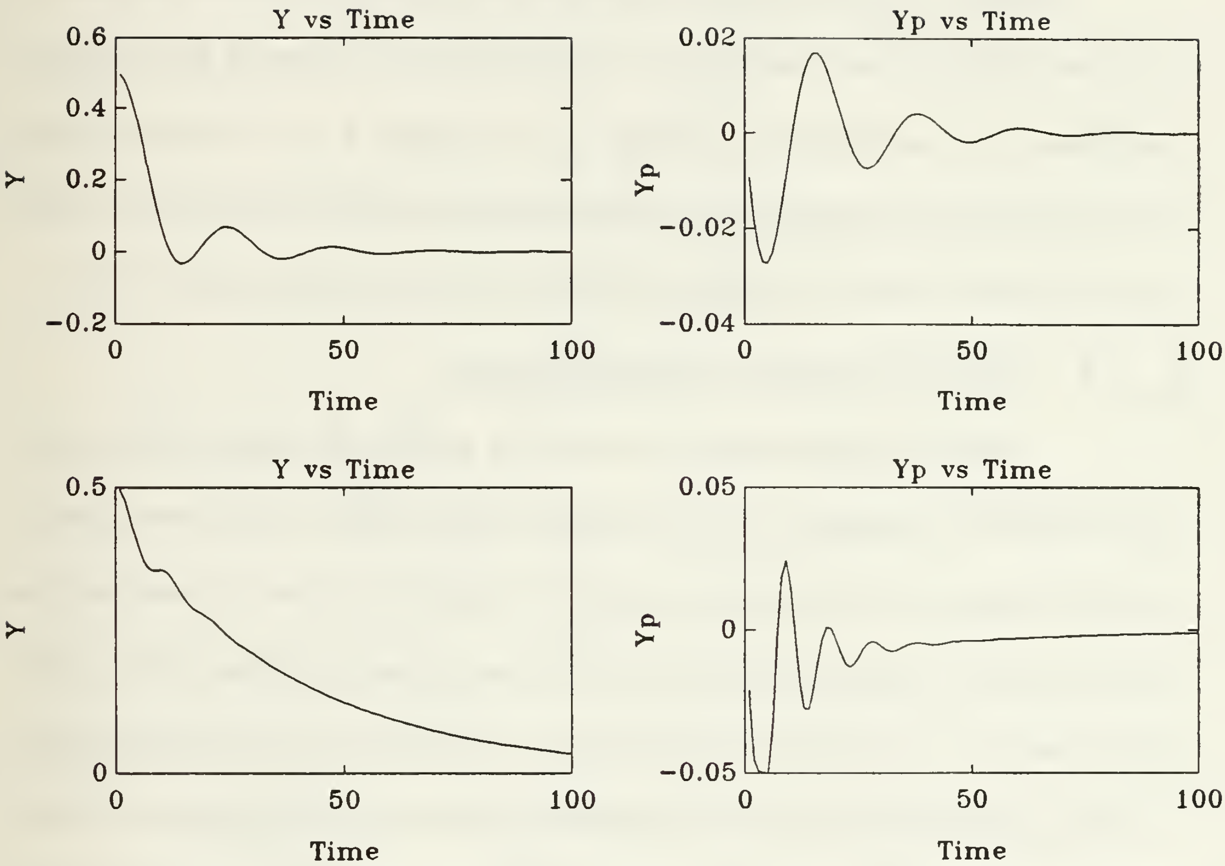

Figure 11. Observable, Non-Observable Comparison of Y, Y_p for η^2=2.0.

This switching speed can be defined in terms of an angle. Note in Figure 12 that as the angle ϕ approaches 90 degrees, the system is being asked to change instantaneously. Another way of saying this is that the system is ordering the towline attachment point to move instantaneously from port to starboard. ϕ=90 degrees is impossible; the designer has to allow the system time to react.

Figure 13 shows the observable and non-observable tanker response of Y, Y_p for ϕ=30, ϕ=45, and ϕ=60. For the observable case, overshoot, the amount of oscillation, and settling time increase as ϕ increases. For the non-observable tanker, the response destabilizes between ϕ=45 and ϕ=60. Clearly there is a limit to the switching speed of the system. ϕ=45 was used for the standard tanker.

6. Figure 14: Variance of LQR Weighting

Figure 14 demonstrates the effect of varying the control effort in the linear quadratic regulator. As α is increased from 0.005 to 0.035, more effort is used to control the towed vessel motion. In Figure 14, Y and Y_p responses are plotted for four different values of α. In all four graphs, the tanker is observable. Note that as α increases, the overshoot in Y_p decreases, and the settling time remains about the same as for the standard tanker. This makes sense; more effort is being expended to control Y_p through Y_{pc}. The response of Y remains essentially unchanged.

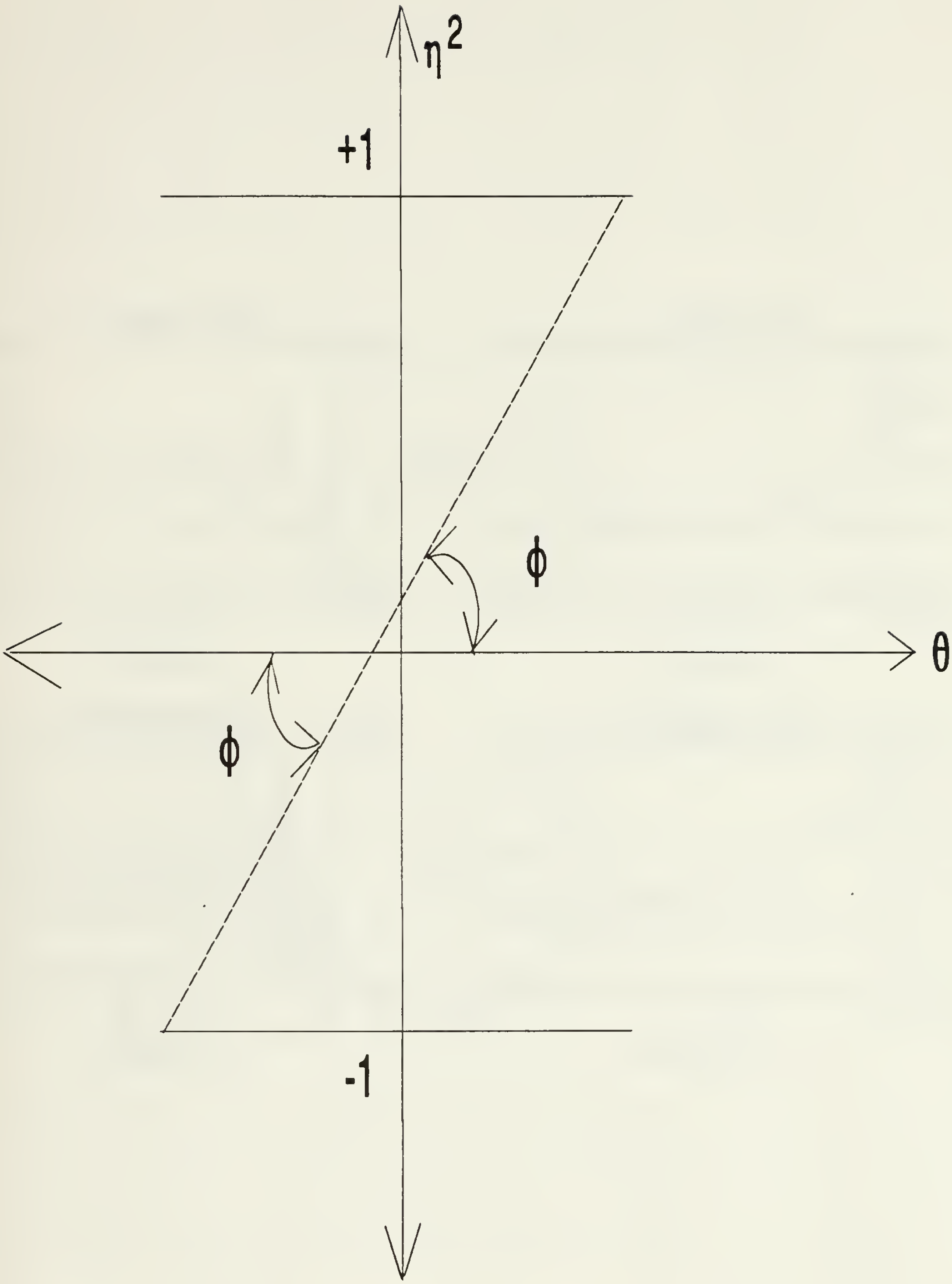

Figure 12. The Sliding Plane.

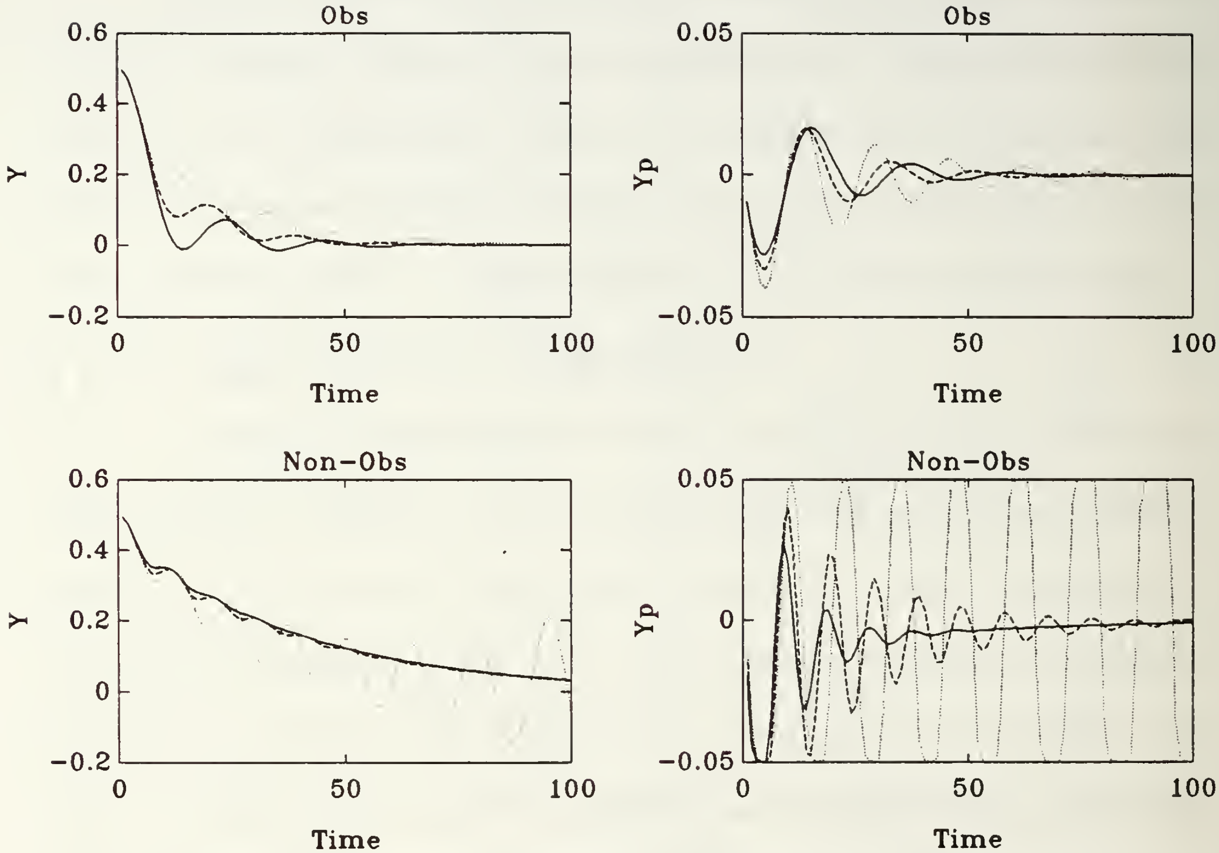

Figure 13. Y, Y_p Variance When Changing Switching Angle θ.
_______ θ= 30°, _ _ _ _ θ= 45° , ... θ = 60°

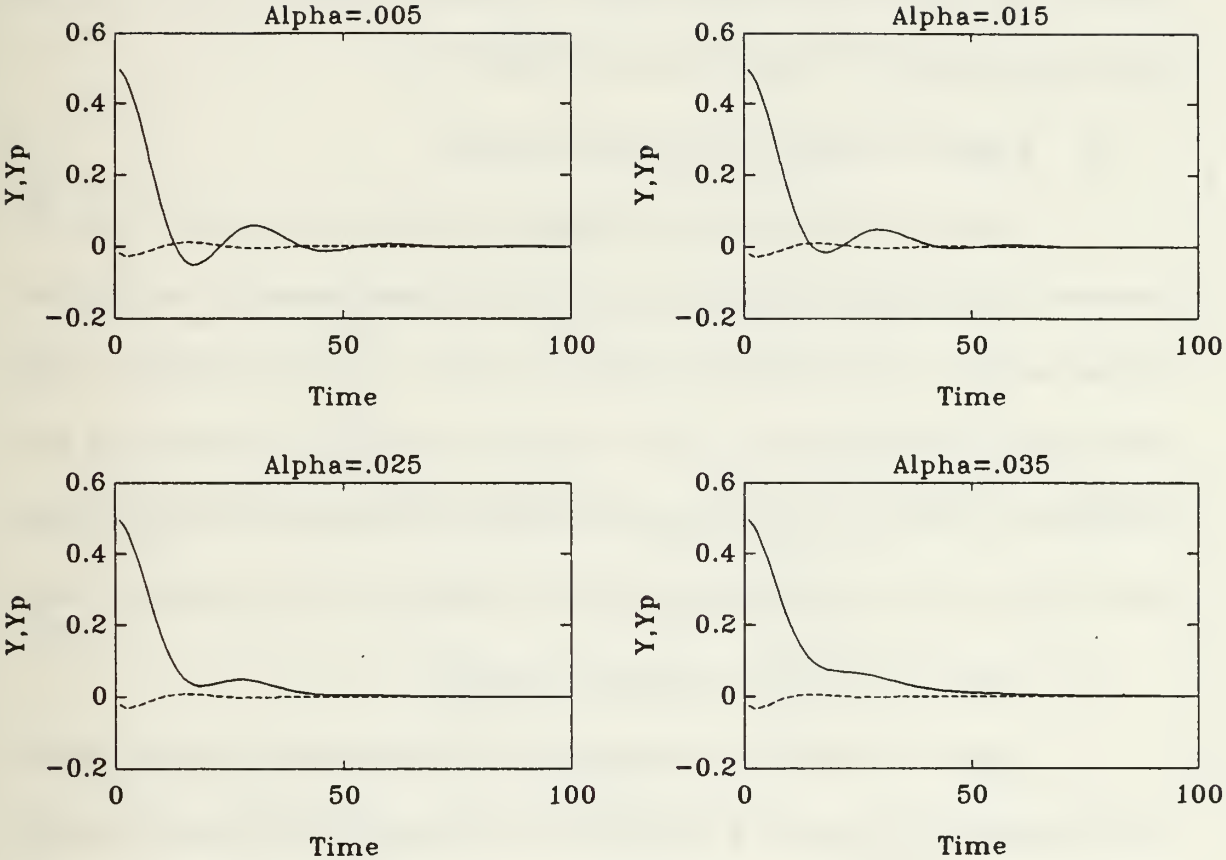

Figure 14. Tanker Response for Y, Y_p as α Varies.
_____ Y_p, _ _ _ Y.

7. Figure 15: Observable Tanker, Variance of T_p

Figure 15 demonstrates the effect of varying the time allotted for Y_p to match Y_{pc} for the standard tanker. The T_p used for the standard case was 0.5. Varying T_p does not significantly affect the behavior unless the change is in order of magnitude. This result has favorable implications; if changing T_p doesn't radically affect the towed vessel response, a smaller motor can be used to drive the device moving the towline attachment point, Y_p.

8. Figure 16: Variance of Towline Length

Figure 16 illustrates how changing the towline length will change the response of the tanker, both in the observable and non-observable case. For this simulation, the non-dimensional towline length, LL, was lengthened and shortened by twenty percent. If the towline is shortened, the overshoot and oscillation in Y and Y_p will increase for both observable and non-observable cases; the reverse is true is the towline is lengthened. Settling time will remain the same.

9. Figure 17: Variance of Towline Tension

Figure 17 shows how changing the non-dimensional towline tension will change tanker response. If the tension in decreased, the overshoot remains about the same; oscillation and settling time in Y and Y_p increase for both the observable and non-observable cases. The reverse is true as the tension is increased. A 20 percent change in towline does not cause a radical change in system response.

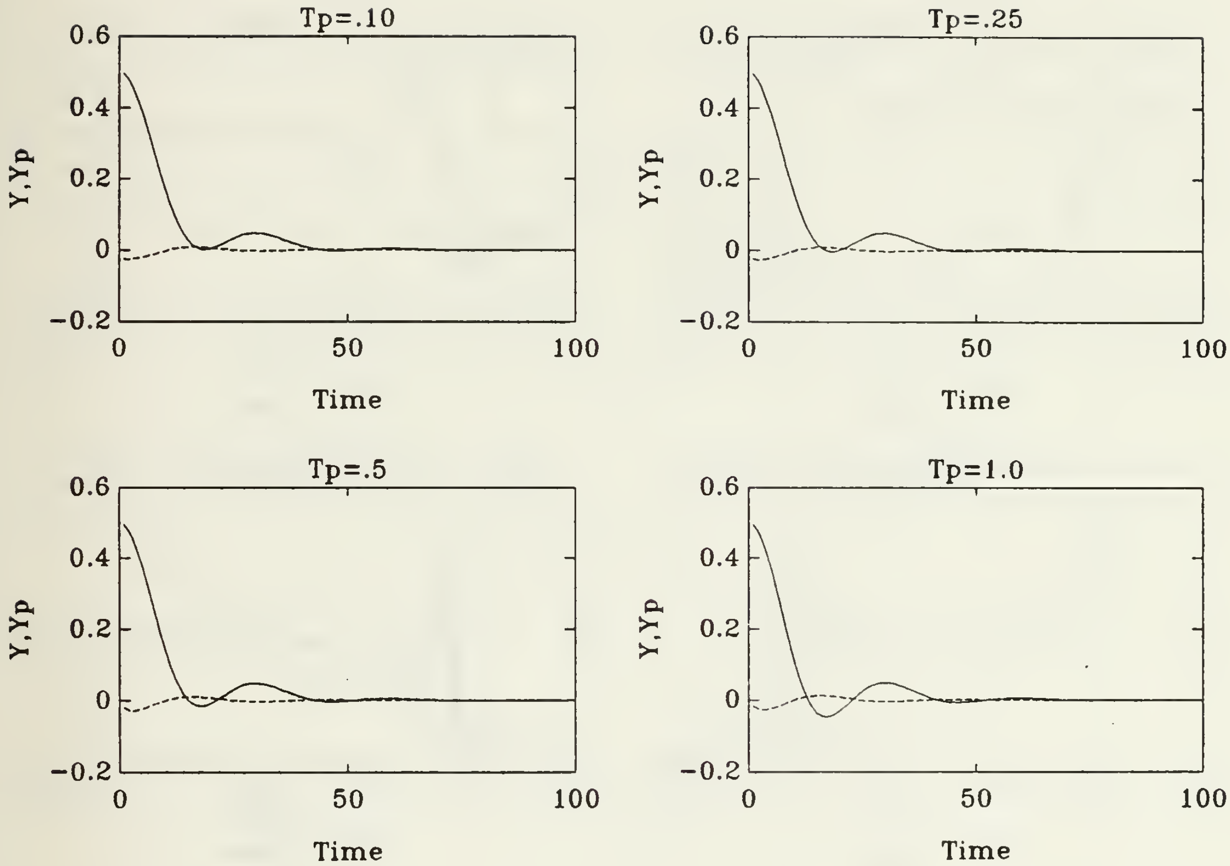

Figure 15. Observable, Non-Observable Response as T_p Varies. ______ y_p, _ _ _ _ y.

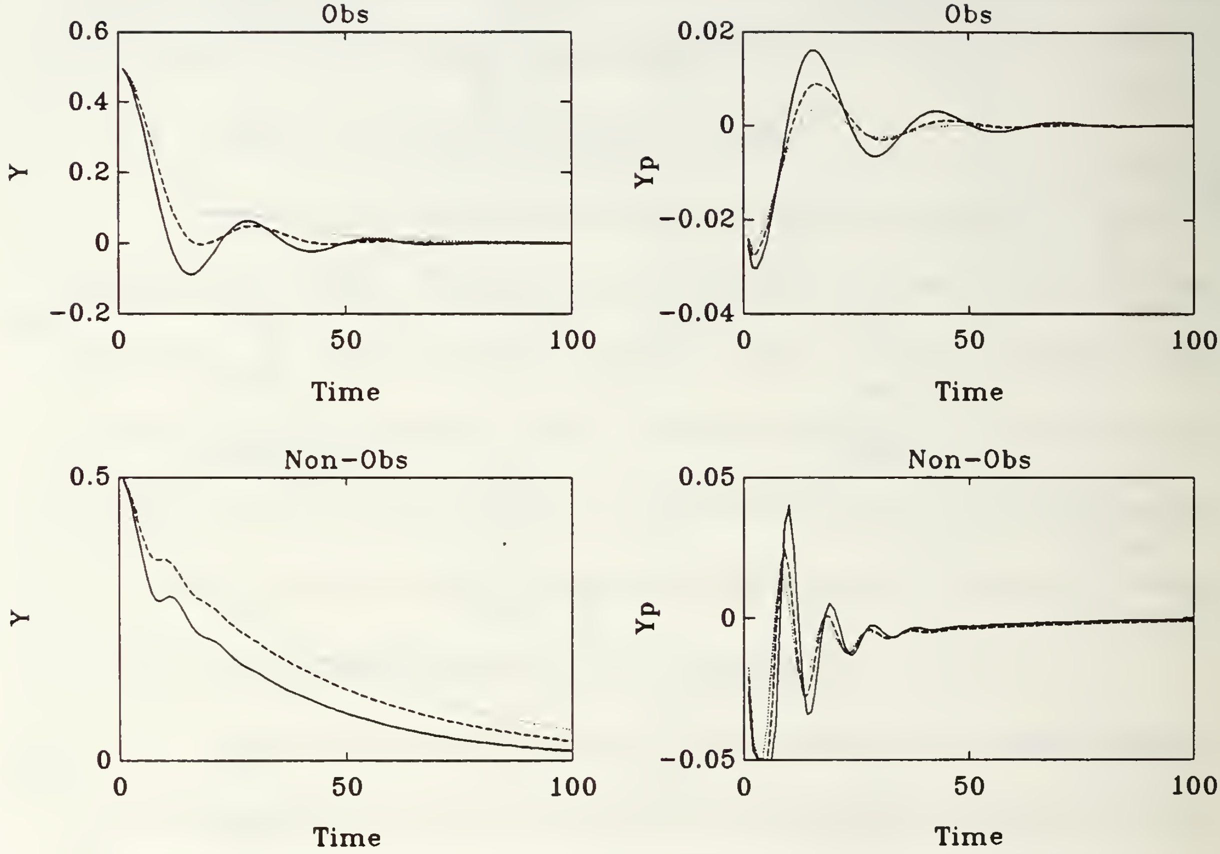

Figure 16. Observable, Non-Observable Response for Variance of LL.
_____ LL=2.0, _ _ _ _ LL=2.5, ... LL=3.0.

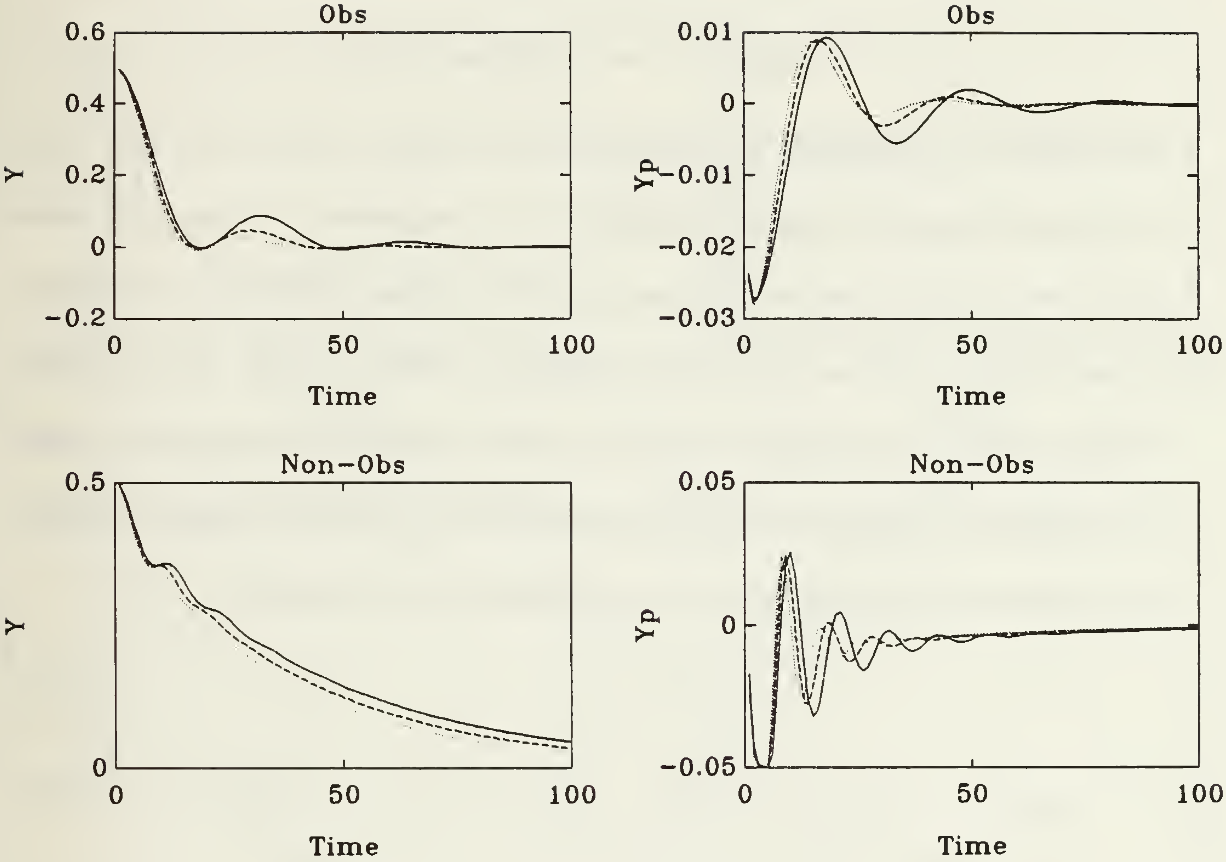

Figure 17. Observable, Non-Observable Tanker: Variance of T.
_____ T=.0008 _ _ _ _ T=.0010 ..... T=.0012

10. Non-Linear (Large Angle) Motion

Recall from page 6 that the horizontal towline force associated in the sway direction is

$$T \sin(\gamma + \psi)$$

and the moment due to the towline causing the vessel to yaw is:

$$-Tx_p(\sin(\gamma + \psi)) - Ty_p \cos(\gamma + \psi)$$

Previously the assumption was made that the range of motion for γ and ψ was less than 30 degrees. This simulation lifts this restriction; the sine and cosine terms from the towline tension term are left in when running the simulation. Figure 18 plots the standard tanker response for both linear and non-linear tensions. Note that the tanker response remains stable; the sliding mode control law is robust enough to handle non-linearities in the towline tensions. The non-linear Fortran simulation program is included in the Appendix.

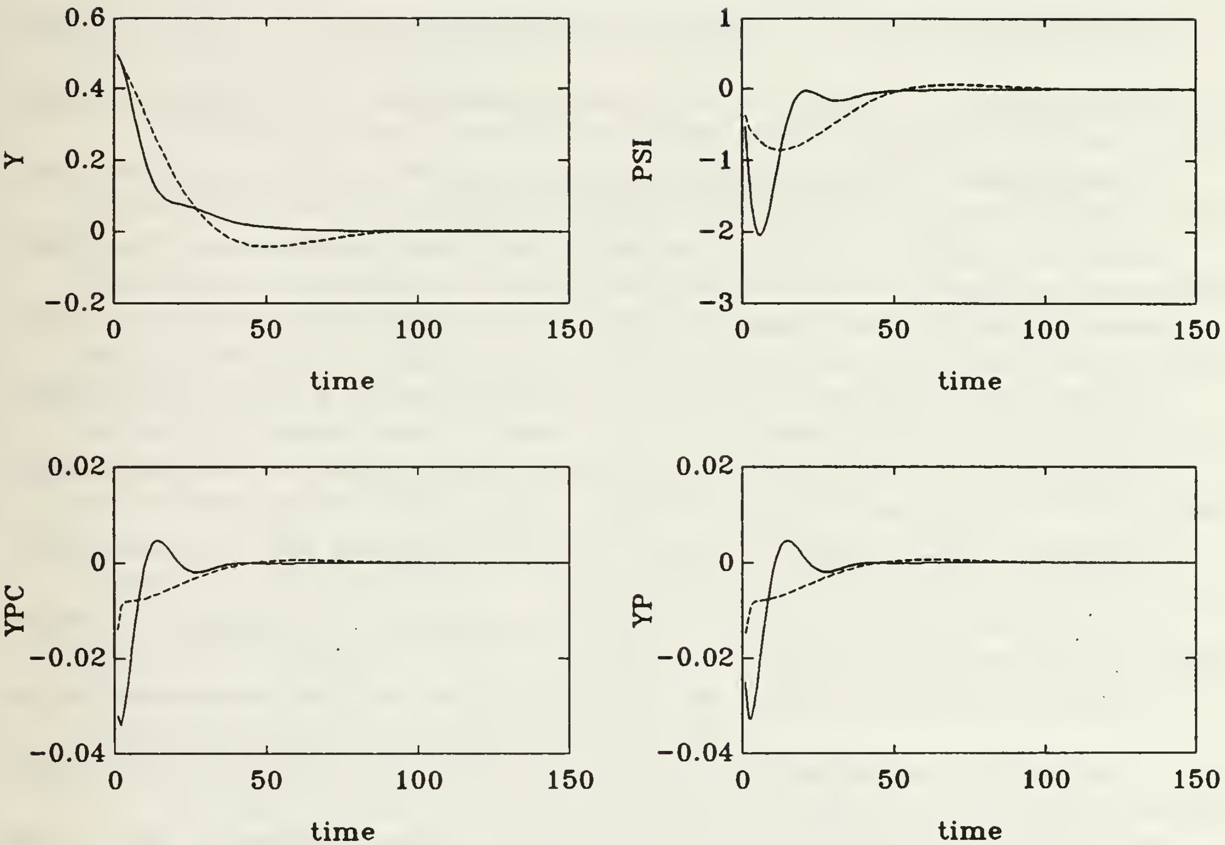

Figure 18. Non-Linear, Linear Tankers With Standard Parameters.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

This thesis explored the possibility of athwartships movement of the towline attachment point as a means of improving the stability of towed vessels. Newell demonstrated that use of a full order observer will work only when the towed vessel trajectory is close to that of the towing vessel [Ref. 5]. Sliding mode control, in conjunction with the Linear Quadratic Regulator cost function, provides a much more robust method of controlling the towed vessel motion. The sliding mode control law can ensure stability of the system, even if not all the state variables can be observed. The only restrictions for the numerical applications considered in this work are that a toggling speed such that $\dot{\phi}$ is less than an angle between 45 and 60 degrees is used, and that the toggling gain η^2 is greater than a number between 0.5 and 1.0. The LQR function allows the designer to choose where his emphasis is placed. By choosing whether to place the emphasis on the control effort, the designer can minimize the size of the motor required to drive the mechanical apparatus used to move the towline attachment point. If the designer wants to minimize steady state error, he can change the LQR function to reflect that emphasis. The LQR function can also be used to minimize specific state variables, such as in the case studied, where ψ was chosen to be minimized. It should be emphasized that a "worst-case" scenario was used to demonstrate the

robustness of the system; a large, unwieldy tanker was used, and two of the four state variables were declared "not observable." Even in the worst case the sliding mode control law performed adequately. Towline tension and length were increased and decreased by 20 percent, and the system remained stable. The small angle motion assumption was dropped, and yet the system continued to perform satisfactorily.

B. RECOMMENDATIONS

The sliding mode control law should be robust enough to cover most of the unmodeled dynamics and non-linearities, but two of the assumptions made at the beginning of the thesis must be investigated further; the massless, inextensible towline, and the no wind, wave, or current (disturbances) assumptions. In reality the towline tension does not remain constant; consequently, the surge velocity u of the towed vessel will also vary. A method to take these aspects of motion into account needs to be looked at; integral control is one possibility. Another is to determine a wave and towline function, and add the two functions to the right side of the sway and yaw equations of motion.

This thesis has shown that it is possible to stabilize and control the motion of a towed vessel with a sliding mode control law that uses linear quadratic regulator gains. Further analysis needs to be done on the effects of disturbances and a time-variant towline tension on the system.

APPENDIX

TABLE I. TOWED VESSEL DATA

Property	Vessel: Barge	Tanker	Mariner
LBP, ft	191.56	1066.3	528
$x'_{\dot{u}}$	-0.00136	-0.0009	-0.0004444
m'	0.170	0.0181	0.00888
$Y'_{\dot{v}}$	-0.01383	-0.0171	-0.00912
Y'_{v}	-0.0153	-0.0261	-0.01434
$Y'_{\dot{\psi}}$	0	0	0
Y'_{ψ}	0.00238	0.00365	0.00456
$N'_{\dot{v}}$	0	0	0
N'_{v}	-0.007285	-0.0105	-0.0046
$I'_{z}-N'_{\dot{\psi}}$	0.00188	0.00222	0.00115
N'_{ψ}	-0.00128	-0.0048	-0.00296

TABLE 2. NONDIMENSIONAL TERMS

$m' = m / m_r$	$Y'_v = Y_v / (m_r u_o / L)$
$u' = u / u_p$	$Y'_{\dot{v}} = Y_{\dot{v}} / m_r$
$v' = v / u_c$	$Y'_r = Y'_\psi = Y_\psi / (m_r u_o)$
$\dot{u}' = \dot{u} / (u_o^2 - L)$	$Y'_{\dot{r}} = Y'_\psi = Y_\psi / (m_r L)$
$\dot{v}' = \dot{v} / (u_c^2 - L)$	$N'_v = N_v / (m_r u_c)$
$\dot{t}' = \dot{t} / (u_c^2 - L)$ (*time*)	$N'_{\dot{v}} = N_{\dot{v}} / (m_r L)$
$I'_z = I_z / (m_r L^2)$	$N'_r = N'_\psi = N_\psi / (m_r u_c L)$
$r' = \psi' = \psi / (u_c / L)$	$N'_{\dot{r}} = N'_\psi = N_\psi / (m_r L^2)$
$\dot{r}' = \ddot{\psi}' = \ddot{\psi} / (u_c^2 / L^2)$	$T' = T / (m_r u_o^2 / L)$ (*Tension*)
$x_g^1 = x_g / 1$	$l' = l / L$
$y_g^1 = y_g / L$	$x'_p = x_p / L$
$X'_u = X_u / (m_r u_c / L')$	$y'_p = y_p / L$
$X'_{\dot{u}} = X_{\dot{u}} / m_r$	

```
Xudot=-0.0009;
M=0.0181;
Yvdot=-0.0171;
Yv=-0.0261;
Yrdot=0.0;
Yr=0.00365;
Ydel=0.0278;
Nvdot=0.0;
Nv=-0.0105;
NrdotIz=-.00222
Nr=-0.0048
Ndel=-0.0139
inquire LL
inquire Xp
inquire T
U=1.0;
L=1.0;
D=NrdotIz*(Yvdot-M)-Nvdot*Yrdot;
a13=1.0;
a24=1.0;
a31=(1/D)*(T/LL)*(NrdotIz-Xp*Yrdot);
a32=(1/D)*((Yv*U+T*(1+Xp/LL))*NrdotIz-Yrdot*(Nv*U+T*Xp*(1+Xp/LL)));
a33=(1/D)*(Nv*Yrdot-Yv*NrdotIz);
a34=(1/D)*((Yvdot*U-Yr)*NrdotIz-Yrdot*(Nvdot*U-Nr));
a41=(1/D)*(T/LL)*(Xp*(Yvdot-M)-Nvdot);
a42=(1/D)*((Yvdot-M)*(Nv*U+T*Xp*(1+Xp/LL))-Nvdot*(Yv*U+T*(1+Xp/LL)));
a43=(1/D)*(Nvdot*Yv-Nv*(Yvdot-M));
a44=(1/D)*(Nvdot*Yr-Nr*Yvdot+(Nr-Nvdot*U)*M);
b3=(1/D)*T*(NrdotIz/LL-Yrdot*(1+Xp/LL));
b4=(1/D)*T*((Yvdot-M)*(1+Xp/LL)-Nvdot/LL);
A=[0,0,a13,0;0,0,0,a24;a31,a32,a33,a34;a41,a42,a43,a44];
B=[0;0;b3;b4];
C=[0,0,1,0];
D=[0];
Q=[0 0 0 0;0 131.3316 0 0;0 0 0 0;0 0 0 0];
inquire alpha
R=[(1/alpha)**2];
<EIG,K>=REGULATOR(A,B,Q,R);
INQUIRE TP
A1=[-1/TP,0,0,0,0;0,0,0,1,0;0,0,0,0,1;B3,A31,A32,A33,A34;B4,A41,A42,A43,A44];
B1=[1/TP;0;0;0;0];
S=[1,K];
G1=-INV(S*B1)*S*A1;
G2=-INV(S*B1);
ETA2=G2
GAIN=G1
SIGMA=S
```

```
C
C
      REAL KYP,K1,K2,K3,K4,L,LL,Nvdot,M,Nv,Nr,Nrdot
      Xudot=-0.0009
      M=0.0181
      Yvdot=-0.0171
      Yv=-0.0261
      Yrdot=0.0
      Yr=0.00365
      Ydel=0.0278
      Nvdot=0.0
      Nv=-0.0105
      Nrdot=-0.00222
      Nr=-0.0048
      Ndel=-0.0139
      U=1.0
      L=1.0
C
      OPEN (10,FILE='LINEAR.DAT',STATUS='OLD')
      OPEN (11,FILE='Y.DAT',STATUS='NEW')
      OPEN (12,FILE='PSI.DAT',STATUS='NEW')
      OPEN (13,FILE='YP.DAT',STATUS='NEW')
      OPEN (14,FILE='YPC.DAT',STATUS='NEW')
C
      READ (10,*) TSIM,DELTAT,IPRNT
      READ (10,*) KYP,K1,K2,K3,K4
      READ (10,*) SYP,S1,S2,S3,S4
      READ (10,*) GBAR
      READ (10,*) TP,LL,XP,T
      READ (10,*) ETA2,PHI
      READ (10,*) Y,PSI,SAT
C
      ISIM=TSIM/DELTAT
      PI=4.0*ATAN(1.0)
      PHI=PHI*PI/180.0
      PSI=PSI*PI/180.0
      SATP= SAT
      SATM=-SAT
      X1=Y
      X2=PSI
      X3=0.0
      X4=0.0
      YP=0.0
      YPC=0.0
C
      D=Nrdot*(Yvdot-M)-Nvdot*Yrdot
      a13=1.0
```

```
      a24=1.0
      a31=(1/D)*(T/LL)*(Nrdot-Xp*Yrdot)
      a32=(1/D)*((Yv*U+T*(1+Xp/LL))*Nrdot-Yrdot*(Nv*U+T*Xp*(1+Xp/LL)))
      a33=(1/D)*(Nv*Yrdot-Yv*Nrdot)
      a34=(1/D)*((Yvdot*U-Yr)*Nrdot-Yrdot*(Nvdot*U-Nr))
      a41=(1/D)*(T/LL)*(Xp*(Yvdot-M)-Nvdot)
      a42=(1/D)*((Yvdot-M)*(Nv*U+T*Xp*(1+Xp/LL))
     &   -Nvdot*(Yv*U+T*(1+Xp/LL)))
      a43=(1/D)*(Nvdot*Yv-Nv*(Yvdot-M))
      a44=(1/D)*(Nvdot*Yr-Nr*Yvdot+(Nr-Nvdot*U)*M)
      b3=(1/D)*T*(Nrdot/LL-Yrdot*(1+Xp/LL))
      b4=(1/D)*T*((Yvdot-M)*(1+Xp/LL)-Nvdot/LL)
C
C     SIMULATION BEGINS
C
      DO 1 I=1,ISIM
        YPDOT =-YP/TP+YPC/TP
        X1DOT = X3
        X2DOT = X4
        X3DOT = B3*YP + A31*X1 + A32*X2 + A33*X3 + A34*X4
        X4DOT = B4*YP + A41*X1 + A42*X2 + A43*X3 + A44*X4
C
        YP = YP + YPDOT*DELTAT
        X1 = X1 + X1DOT*DELTAT
        X2 = X2 + X2DOT*DELTAT
        X3 = X3 + X3DOT*DELTAT
        X4 = X4 + X4DOT*DELTAT
C
C       CONTROL LAW
C
        SIGMA = SYP*YP + S1*X1 + S2*X2 + S3*X3 +S4*X4
        SATSGN= SIGMA/TAN(PHI)
        IF (SATSGN.GE.(+1.0)) SATSGN=+1.0
        IF (SATSGN.LE.(-1.0)) SATSGN=-1.0
        UHAT  = KYP*YP + K1*X1 + K2*X2 + K3*X3 + K4*X4
        UBAR  = ETA2*GBAR*SATSGN
        YPC   = UHAT+UBAR
        IF (YPC.GE.SATP) YPC=SATP
        IF (YPC.LE.SATM) YPC=SATM
C
        Y  =X1
        PSI=X2
C
C       PRINT RESULTS
C
        J=J+1
        IF (J.NE.IPRNT) GO TO 1
        TIME=I*DELTAT
        WRITE (*,*) TIME,Y
        WRITE (11,*) TIME,Y
```

```
      WRITE (12,*) TIME,PSI*180.0/PI
      WRITE (13,*) TIME,YP
      WRITE (14,*) TIME,YPC
      J=0
 1  CONTINUE
    STOP
    END
```

```
C     NONLINEAR SIMULATION - TANKER
C     SURGE NOT INCLUDED
C
      REAL KYP,K1,K2,K3,K4,L,LL,NVDOT,NV,IZ,NRDOT,NR,NVRR
C
      XUDOT=-0.0009
      M     = 0.0181
      YVDOT=-0.0171
      YV    =-0.0261
      YRDOT= 0.0
      YR    = 0.00365
      NVDOT= 0.0
      NV    =-0.0105
      IZ    = 0.0
      NRDOT=-0.0022
      NR    =-0.0048
      YVRR =-0.045
      NVRR = 0.0061
      UTOW = 10.0
      L     = 1066.3
      RHO  = 1.9905
      SB    = 680625.0
      P     = 9.78
      Q     = 1.93
      SB    = SB/(0.5*RHO*L*L*UTOW*UTOW)
C
      OPEN (10,FILE='LINEAR.DAT',STATUS='OLD')
      OPEN (11,FILE='Y.RES',STATUS='NEW')
      OPEN (12,FILE='PSI.RES',STATUS='NEW')
      OPEN (13,FILE='YP.RES',STATUS='NEW')
      OPEN (14,FILE='YPC.RES',STATUS='NEW')
      OPEN (15,FILE='NONLINEAR1.RES',STATUS='NEW')
C
      READ (10,*) TSIM,DELTAT,IPRNT
      READ (10,*) KYP,K1,K2,K3,K4
      READ (10,*) SYP,S1,S2,S3,S4
      READ (10,*) GBAR
      READ (10,*) TP,LL,XP,T
      READ (10,*) ETA2,PHI
      READ (10,*) Y,PSI,SAT
C
      ISIM = TSIM/DELTAT
      PI    = 4.0*ATAN(1.0)
      PHI   = PHI*PI/180.0
      PSI   = PSI*PI/180.0
      SATP = SAT
      SATM =-SAT
```

```
      X    = LL-XP
      U    = 1.0
      V    = 0.0
      R    = 0.0
      YP   = 0.0
      YPC  = 0.0
C
      DEN=(M-YVDOT)*(IZ-NRDOT)-YRDOT*NVDOT
C
C      SIMULATION BEGINS
C
      DO 1 I=1,ISIM
C
        F2=YV*V+0.5*YVRR*V*R*R+(YR-M)*R
        F3=NV*V+0.5*NVRR*V*R*R+NR*R
C
        SING=(Y+XP*SIN(PSI)+YP*COS(PSI))/LL
        GAMMA=ASIN(SING)
C
        YPDOT =-YP/TP+YPC/TP
        VDOT  =(F2-T*SIN(GAMMA+PSI))/(M-YVDOT)
        RDOT =(F3-T*XP*SIN(GAMMA+PSI)-T*YP*COS(GAMMA+PSI))/(IZ-NRDOT)
        YDOT  =U*SIN(PSI)+V*COS(PSI)
        PSIDOT=R
C
        YP  = YP  + YPDOT *DELTAT
        U   = U   + UDOT  *DELTAT
        V   = V   + VDOT  *DELTAT
        R   = R   + RDOT  *DELTAT
        X   = X   + XDOT  *DELTAT
        Y   = Y   + YDOT  *DELTAT
        PSI = PSI + PSIDOT*DELTAT
C
C       CONTROL LAW
C
        SIGMA = SYP*YP + S1*Y + S2*PSI + S3*YDOT + S4*PSIDOT
        SATSGN= SIGMA/TAN(PHI)
        IF (SATSGN.GE.(+1.0)) SATSGN=+1.0
        IF (SATSGN.LE.(-1.0)) SATSGN=-1.0
        UHAT  = KYP*YP + K1*Y + K2*PSI + K3*YDOT + K4*PSIDOT
        UBAR  = ETA2*GBAR*SATSGN
        YPC   = UHAT+UBAR
        IF (YPC.GE.SATP) YPC=SATP
        IF (YPC.LE.SATM) YPC=SATM
C
C       PRINT RESULTS
C
        J=J+1
        IF (J.NE.IPRNT) GO TO 1
        TIME=I*DELTAT
```

```
      WRITE (*,*) TIME,Y
      WRITE (11,*) TIME,Y
      WRITE (12,*) TIME,PSI*180.0/PI
      WRITE (13,*) TIME,YP
      WRITE (14,*) TIME,YPC
      WRITE (15,*) TIME,Y,PSI*180.0/PI,YP,YPC
      J=0
1   CONTINUE
    STOP
    END
```

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